Letters

Comments on "A Procedure for Calculating Fields Inside Arbitrarily Shaped, Inhomogeneous Dielectric Bodies Using Linear Basis Functions with the Moment Method"

MARK J. HAGMANN

The above paper[1] recently described the use of a linear basis with polyhedral cells, which is a significant modification from previous block model solutions [1] which have used a pulse-function basis (PFB) with cubical cells (PFBC). It is reasonable to expect greater accuracy when the same number of cells is used with the new procedure, but some of the comments regarding earlier PFBC models may be misleading.

Tsai *et al.*[1] state that earlier studies of PFBC models [2] showed "serious deficiencies" in that subdivision of cubical cells caused divergence, but they do not mention that there were objections to their use of subdivision [3]. It is essential that an array of cells be a best fit of the object being modeled [1]. Whenever a cell on the boundary is subdivided, if that cell contains a corner or edge not present in the object, then the solution will not represent the object and may tend to diverge. It appears that the examples given by Tsai *et al.* with their new procedure use only best-fit arrangements of cells. Subdivison may cause less of a problem with polyhedral cells, since the angles at corners and edges tend to be more blunt than with cubes, but the authors should use the same standards when they test the two procedures.

Tsai *et al.* have changed both the basis and the cell shape relative to those used in PFBC models. Interpretation would be simpler if these two changes were made separately. It is unlikely that the linear basis (emphasized in the title) would give improved accuracy in solutions for smooth objects if it were not combined with the use of polyhedral cells. I anticipate that in quasi-static problems there would be little loss of accuracy if the polyhedral cells were used with a PFB. Furthermore, it is not clear that the accuracy at higher frequencies is increased by using a linear basis in place of a PFB (with the same total number of unknowns) when polyhedral cells are used.

PFBC models have sufficient accuracy for many applications [1]. For example, the accuracy of SAR values on the axis of a PFBC model with a single column of six cubes having different sizes would be comparable to that of the values which Tsai *et al.* gave in Fig. 8 for their model of a medium-size rat. I would be happy to make a comparison if the author would supply more details regarding the shape that was modeled and their calculations for several frequencies. In problems for which the electric field is slowly varying within each cell, the principal source of error with PFBC models appears to be the imperfect fit of boundaries by the cubical cells, and not the PFB. This error may be mitigated by using one of several procedures (e.g. fitting the boundaries with cubes of several sizes). Polyhedral cells simplify the task of approximating smooth surfaces, but are likely to require a significant increase in CPU time, even without a linear basis.

Reply[2] *by C.-T. Tsai, H. Massoudi, C. H. Durney, and M. F. Iskander*[3]

We have conjectured that the inaccuracies in the calculation of internal field distribution using the moment method with pulse basis functions and cubical cells (PFBC) are caused by the inadequacies of the pulse basis functions in satisfying the boundary conditions at dielectric discontinuities and therefore in representing surface charge density, which we have found critically affects numerical results. For example, quoting from [3, last paragraph on p. 349]:

> Since a pulse function has only one value in a cell, the boundary conditions at both surfaces could not be satisfied well at all, even in an approximate sense. Even when a large number of cells is used, the boundary conditions at interfaces between adjacent cells of different permittivities would not be satisfied very well by pulse functions. We believe that the boundary conditions are very important to the accuracy of the solution because of the surface charge density induced at a discontinuity in permittivity. As we mentioned above, our results lead us to believe that the accuracy of the numerical solution of the integral equation depends strongly on the adequacy with which the surface charge density is accounted for in the numerical solution. If this is true, then the limitation of the pulse functions in satisfying boundary conditions, and therefore in describing surface charge density at permittivity discontinuities, may be the main reason for numerical inaccuracies.

Since our paper was published, a much clearer understanding of the problem has been provided by Borup *et al.* [4]. They showed that the inaccuracies in calculations using the PFBC can indeed be attributed to inadequate representation of surface charge density in terms of two factors: (1) inadequate geometric modeling of the dielectric interfaces by the cubical cells and (2) inability of the pulse functions to satisfy the boundary conditions at the cell walls, which produces an incorrect surface charge density that causes errors in the calculations.

Borup *et al.* showed this by making calculations in dielectric cylinders for the TE case. First, they modeled the dielectric discontinuities with polyhedral cells instead of cubical cells and calculated internal field distributions using pulse basis functions (one of Dr. Hagmann's suggestions). The results indicated that significant errors still occurred. They attributed these errors to fictitious surface charge density produced by the discontinuity of the normal electric field pulse basis functions at the boundaries

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[1] C.-T Tsai, H. Massoudi, C. H. Durney, and M. F. Iskander, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 1131–1139, Nov. 1986.

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[2] Manuscript received April 24, 1987

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between two cells of equal permittivity. Although the normal component of the actual electric field at the boundary between two cells of equal permittivity must be continuous, the discontinuity of the pulse functions at the boundary of the two cells is unavoidable because the pulse functions are constant throughout the cells. The fictitious surface charge density that is equivalent to the discontinuity in the pulse functions acts as a source of calculated fields that should actually be zero.

Next, they made PFBC calculations, but removed the fictitious surface charge density by simply not integrating the integral surface charge density term over any surfaces between cells having the same permittivity. Again, they found serious errors in the results. Then they both removed the fictitious surface charge density and used polyhedral cells to model the surface discontinuities more accurately. In this case, they found good agreement between numerical calculations, both for a homogeneous cylinder and for a two-layer cylinder. It is important to note the advantage of using the free-space Green's function integral equation (FGIE), which contains an explicit source term for the surface charge density. Since the dyadic Green's function integral equation (DGIE) does not contain a term that specifically corresponds to the charge density, elimination of the fictitious surface charge density would not be tractable with the DGIE. Also, as we pointed out [3], the FGIE gave more accurate results for our calculations with pulse basis functions and cubical cells than the DGIE. We attributed this to the sensitivity of the calculations to the charge density source term.

In our opinion, the results of Borup *et al.* [4] clearly demonstrate that the combination of the inadequate representation of the surfaces between dielectric discontinuities by the cubical cells and the inability of the pulse basis functions to satisfy the boundary conditions between cells is the primary source of error in the PFBC numerical calculation of internal field distribution. It seems clear that satisfactory calculations using an integral equation formulation will therefore require modeling dielectric discontinuities by polyhedral cells, even though this is significantly more complicated than using cubical cells.

An interesting question that should be investigated is whether using linear basis functions with polyhedral cells would require fewer unknowns than using pulse basis functions with polyhedral cells. Since linear basis functions can represent fields inside cells, including boundary conditions, much better than pulse functions, we found that larger cells could be used with linear basis functions than with pulse functions, and in the cases we tested, we obtained better accuracy with linear basis functions and polyhedral cells than with pulse basis functions and polyhedral cells for the same number of unknowns [5]. If the number of unknowns using linear basis functions were reduced by relating the fields in adjacent cells through the boundary conditions, as suggested in the paper in question, it might be possible to get better accuracy with fewer unknowns using linear basis functions.

References

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[4] D T. Borup, D M Sullivan, and O. P. Gandhi, "Comparison of the FFT conjugate gradient method and the finite-difference time-domain method for the 2-D absorption problem," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, pp. 383–395, Apr 1987.

[5] C. T. Tsai, "Numerical calculations of field distributions in 3-D, arbitrary shaped inhomogeneous dielectric objects using the method of moments with linear basis functions," Ph.D. dissertation, Elec. Eng. Dept., Univ. of Utah, 1985.

Corrections to "Spectral-Domain Analysis of Scattering from *E*-Plane Circuit Elements"

QIU ZHANG AND TATSUO ITOH

In the above paper,[1] the expressions for LSM modes should have read as follows:

$$+\begin{cases} \displaystyle\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} D_{mn}^{-}\psi_m(x)\cos(\alpha_n y)\, e^{j\beta'_{mn}(z+W/2)} & z<-W/2 \\ \displaystyle\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \psi_m(x)\cos(\alpha_n y)\left(F_{mn}^{-}e^{j\beta'_{mn}z}+F_{mn}^{+}e^{-j\beta'_{mn}z}\right) & |z|<W/2 \\ \displaystyle\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} D_{mn}^{+}\psi_m(x)\cos(\alpha_n y)\, e^{-j\beta'_{mn}(z-w/2)} & z>W/2 \end{cases} \quad \text{(in eq. (1))}$$

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[1] Q. Zhang and T. Itoh, *IEEE Trans Microwave Theory Tech*, vol. MTT-35, pp. 138–150, Feb. 1987